Preface

This volume contains revised papers that were presented at the international workshop entitled *Computational Methods for Algebraic Spline Surfaces* ("COMPASS"), which was held from September 29 to October 3, 2003, at Schloß Weinberg, Kefermarkt (Austria).

The workshop was mainly devoted to approximate algebraic geometry and its applications. The organizers wanted to emphasize the novel idea of approximate implicitization, that has strengthened the existing link between CAD / CAGD (Computer Aided Geometric Design) and classical algebraic geometry. The existing methods for exact implicitization (i.e., for conversion from the parametric to an implicit representation of a curve or surface) require exact arithmetic and are too slow and too expensive for industrial use. Thus the duality of an implicit representation and a parametric representation is only used for low degree algebraic surfaces such as planes, spheres, cylinders, cones and toroidal surfaces. On the other hand, this duality is a very useful tool for developing efficient algorithms. Approximate implicitization makes this duality available for *general* curves and surfaces.

The traditional exact implicitization of parametric surfaces produce global representations, which are exact everywhere. The surface patches used in CAD, however, are always defined within a small box only; they are obtained for a bounded parameter domain (typically a rectangle, or – in the case of "trimmed" surface patches – a subset of a rectangle). Consequently, a globally exact representation is not really needed in practice. Instead of a single exact high–degree implicit representation, the methods of approximate implicitization produce piecewise implicit surfaces of relatively low degree, which may cover the shape with any desired accuracy. This results in so–called *algebraic spline surfaces*, which can be expected to replace the exact implicit representation in many algorithms.

Compared to the traditional parametric representations, such as rational curves and surfaces (so–called Non-Uniform-Rational-B-Splines – NURBS), algebraic spline surfaces offer several computational advantages. For instance, by exploiting the duality between implicit and parametric representation, the intersection of two surfaces can easily be traced if one of the surfaces is given in implicit, and the other surface is given in parametric form. In this case, the problem can be reduced to a two-dimensional root-finding problem. In the case of two parametric surfaces, one has to solve a four–dimensional problem instead. As another advantage, the fitting of surfaces to scattered data, which is a fundamental tool for generating free-form geometry from prototypes, can be done without mapping the data into a plane – a process which often limits the flexibility and usefulness of the surface fitting techniques which are available today. We also foresee a number of other applications, e.g., in the computer game industry, virtual reality, medical imaging, and scientific computing.

The workshop, and the papers collected in this volume, was devoted both to the theoretical fundamentals and to the various computational aspects which arise in applications of approximate algebraic geometry. These applications are based on techniques developed in different branches of mathematics and computer science, including numerical analysis and scientific computing, algebraic geometry, applied geometry, and computer graphics. For instance, numerical methods are needed to efficiently generate implicit representations, and algebraic techniques are essential for detecting and analyzing singularities, which may help to solve practical problems arising in applied geometry and computer graphics.

Traditionally, these fields are represented by several fairly disjoint scientific communities, which traditionally do not communicate much. In order to stimulate the exchange of ideas, and to promote interdisciplinary research, the workshop brought together experts from the various fields involved.

The papers included in this volume provide an overview about the state-of-the-art in approximative implicitization and various related topics, including both the theoretical basis and the existing computational techniques. This can be expected to encourage and promote the use of approximate implicitization for solving geometric problems in computer-aided design. In some of the papers and in the panel discussion at COMPASS, which is also documented in this volume, the authors try to identify a number of problems (both theoretical and practical ones) which need to be addressed by the different research communities, in order to exploit the potential of implicit representations.

The editors are convinced that this volume will support the mutual exchange of ideas between the various research communities, promoting interdisciplinary research. The interactions between different mathematical disciplines such as approximation theory, classical algebraic geometry and computer aided geometric design will play an essential role for exploiting the new idea of approximate algebraic geometry.

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